

Problem

134. Gas Station(Medium)

There are N gas stations along a circular route, where the amount of gas at station i is $gas[i]$.

You have a car with an unlimited gas tank and it costs $cost[i]$ of gas to travel from station i to its next station $(i+1)$. You begin the journey with an empty tank at one of the gas stations.

Return the starting gas station's index if you can travel around the circuit once in the clockwise direction, otherwise return -1 .

Note:

- If there exists a solution, it is guaranteed to be unique.
- Both input arrays are non-empty and have the same length.
- Each element in the input arrays is a non-negative integer.

Example 1:

Input:

$gas = [1,2,3,4,5]$

$cost = [3,4,5,1,2]$

Output: 3

Explanation:

Start at station 3 (index 3) and fill up with 4 unit of gas. Your tank = $0 + 4 = 4$

Travel to station 4. Your tank = $4 - 1 + 5 = 8$

Travel to station 0. Your tank = $8 - 2 + 1 = 7$

Travel to station 1. Your tank = $7 - 3 + 2 = 6$

Travel to station 2. Your tank = $6 - 4 + 3 = 5$

Travel to station 3. The cost is 5. Your gas is just enough to travel back to station 3.

Therefore, return 3 as the starting index.

Example 2:

Input:

gas = [2,3,4]

cost = [3,4,3]

Output: -1

Explanation:

You can't start at station 0 or 1, as there is not enough gas to travel to the next station.

Let's start at station 2 and fill up with 4 unit of gas. Your tank = 0 + 4 = 4

Travel to station 0. Your tank = 4 - 3 + 2 = 3

Travel to station 1. Your tank = 3 - 3 + 3 = 3

You cannot travel back to station 2, as it requires 4 unit of gas but you only have 3.

Therefore, you can't travel around the circuit once no matter where you start.

Solution

O(n) time, O(1) space

首先还是对原问题进行观察, 是否可以reduce, 可以将原问题reduce成n个子问题: 从第i个gas出发, 是否可以环绕一圈?

这样reduce, 求解过程变为了枚举, 对于枚举而言, 是否有一些**启发式策略**可以减少枚举所需步骤, 一个很直观的策略: 对于第i个地点, 当且仅当 $gas[i] - cost[i] \geq 0$ 时, 第i个地点可以作为出发点. 这样在平均情况下好于暴力求解, 但在最坏情况下, 仍然是 $O(n^2)$ 的算法

这时考虑根据数学原理, 可以得出结论:

如果一个数组的总和是非负, 那么一定可以找到一个起始位置, 从他开始绕数组一圈, 累加和一直都是非负的

那么对于原问题是否有解, 有了很简单的判断方法: 累加 $gas[i] - cost[i]$, 若结果**大于或等于0**, 则一定有解, 问题变成了如何在有解的情况下找到起点

注意我们之前的枚举策略: 假设我们从 $i=0$ 出发, 累加 $sum += gas[i] - cost[i]$, 当遍历到 j 时, $sum < 0$, 表明从0出发是不可行的, 是否应当从 $i+1$ 开始出发, 显然不行:

- 因为 $gas[0]-cost[0] \geq 0$, 则从1开始累加 $gas[i] - cost[i]$ 的结果仍然 < 0 。
- 从0到 j 作为出发点, 同理, 所以可以**直接从 $j+1$ 出发**

[GitHub传送门](#)

```
class Solution
{
public:
    int canCompleteCircuit(vector<int> &gas, vector<int> &cost)
    {
        int sum = 0;
        int index = 0;
        int before = 0;
        for (int i = 0; i < gas.size(); ++i)
        {
            sum += gas[i] - cost[i];
            if (sum < 0)
            {
                before += sum;
                sum = 0;
                index = i + 1;
            }
        }
        if (before + sum < 0)
            return -1;
        return index;
    }
};
```